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Solidification Modeling of Continuous Casting Process

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The aim of the present work was to utilize a new systematic mathematical-informational approach based on informational macrodynamics (IMD) to model and optimize the casting process, taking as an example horizontal continuous casting (HCC). The IMD model takes into account the interrelated thermal, diffusion, kinetic, hydrodynamic, and mechanical effects that are essential for the given casting process. The optimum technological process parameters are determined by the simultaneous solution of problems of identification and optimal control. The control functions of the synthesized optimal model are found from the extremum of the entropy functional having a particular sense of an integrated assessment of the continuous cast bar physicochemical properties. For the physical system considered, the IMD structures of the optimal model are connected with controllable equations of nonequilibrium thermodynamics. This approach was applied to the HCC of ductile iron, and the results were compared with experimental data and numerical simulation. Good agreement was confirmed between the predicted and practical data, as well as between new and traditional methods.

Keywords continuous casting, ductile iron, informational macrodynamics, optimization, solidification modeling

1. Introduction

The traditional modeling of the continuous casting process uses mathematical models and approaches that mostly describe the phenomena of a separated process or severely cross phenomena, simulating mold filling and solidification (Ref 1, 2). These models need massive computations and are not suitable for operative process control. The proposed method is based on informational macrodynamics (IMD), and is used here for systemic informational solidification modeling and the optimization (Ref 3, 4) of the horizontal continuous casting (HCC) of ductile iron (DI).

Horizontal continuous casting is a relatively new, but promising, method of producing near-net shape, high-quality, ferrous cast products (e.g., gray, ductile, and Ni-resist irons and steel), as well as nonferrous alloys (e.g., aluminum and copper) (Ref 5, 6). Figure 1 presents a schematic view of HCC. In this method, liquid metal from the transfer ladle is tapped into the metal receiver. A water-cooled graphite or copper die is attached to the side of the receiver, and a bar is pulled out by an extraction system, which controls stroke length and frequency. A special mechanism cuts and breaks the bars to required lengths. The major advantage of this process is a 92% to 95% casting yield, because it eliminates traditional feeder needs. Liquid metal in the receiver plays the role of a preheated riser that continuously supplies liquid metal to feed the bar during solidification. By maintaining an adequate balance among the metal chemistry, temperature, melt level in the receiver, and the drawing and cooling parameters, it is possible to produce defect-free, high-quality continuous-cast bars (Ref 7).

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2. Mathematical Model of Continuous Casting

The IMD model of HCC takes into account the interrelated thermal, diffusion, kinetic, hydrodynamic, and mechanical effects that are essential for a given casting process. The optimum technological process parameters are determined by the simultaneous solution of problems of identification and optimal control. The control functions of the synthesized optimal model are found from the extremum of the entropy functional (Ref 8). For the physical system considered, the IMD structures of the optimal model are connected with controllable equations of nonequilibrium thermodynamics (NT). Table 1 shows a scheme of the main interrelated physical phenomena that participate in the HCC of DI in which each of the phenomena in this scheme is described in independent phase-variable coordinates (x^t_{n-k}) and spatial coordinates (x^l_{n-k}), where n is the total number of phenomena considered in the model and k is the number of the phenomenon interpreted from the origin (in this case, heat conduction).

All phase coordinates have the minimal number of state variables, which characterize the HCC process, and have the meaning of corresponding derivatives and integrals regarding temperature θ, and the associated physical quantities: thermomechanical stress (*TS*); rate of solidification (v); concentration increment ($\Delta C'$); rate of mass transfer (v_m); thermodynamic stress (σ), strain (ε), density (ρ) , and pressure (p). Each coordinate x_{n-k} in Table 1 is presented as a derivative from coordinate x^t_{n-k+1}, where the total number of variables n of the specific mathematical model is previously unknown. For example, to describe heat conduction (*T*) and solidification (*S*), the Fourier and Stefan equations are used. Diffusion (*D*) is described by equations of Fick's first and second laws, which are associated with the laws of conservation. The equations of *TS* are determined by the bar temperature distribution in time and space, for instance, by the solution of equations *T* and the equations that describe the phase conversions associated with the change in the concentration of carbon, that is, $\Delta C' = C' - C1$ in the liquid phase (C') and the solid phase ($C1$). To describe the hydrodynamics (*H*), the Navier-Stokes equation is

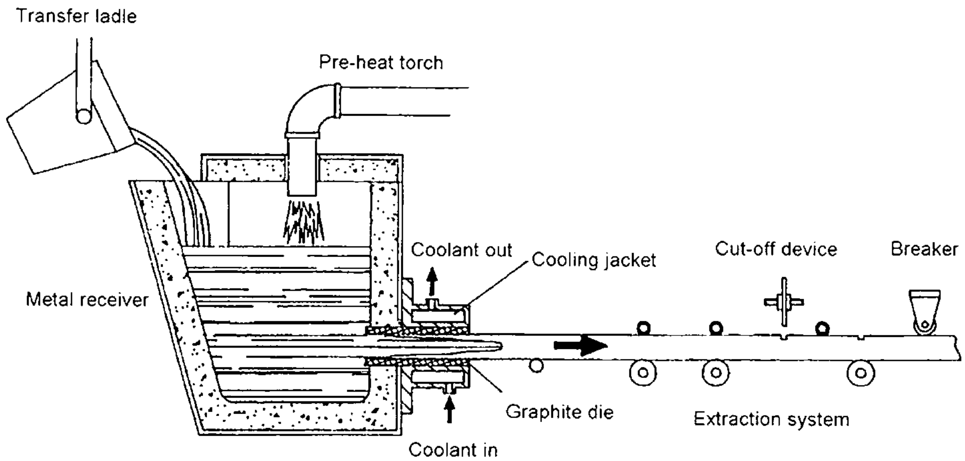

Fig. 1 Schematic of the HCC process

used, and so forth for all variables. Also, in Table 1 M is the mass transfer initiated by effective diffusion and HT is the motion of the liquid phase under the influence of the hydrodynamic forces.

Finally, the systematic mathematical model of the HCC includes the informational form of the NT equations, which together with the main phenomena reflect the effect of overlapping phenomena. The resulting system of n differential equations of the first order (with variables shown in Table 1) forms the systematic macro-model of the HCC, which also describes the dynamics of consolidation of the interactive processes taking place during casting.

As an optimization criterion for the systematic macro-model, the entropy functional of quality is employed, the minimum value of which determines the maximum order of the structure of the bar. This criterion, which is determined by the conditions of the extremum of the nonequilibrium entropy functional for the process (Table 1), emerges as a general indicator of bar structural uniformity on the micro and macro levels. The order on the micro level implies the structural uniformity of the metallic matrix through the cross section of the continuous cast bar with uniform distribution of graphite nodules, and the uniform distribution of chemical elements through the cross section of the grain boundaries without segregation. The order on the macro level corresponds to the production of the same bar with a uniform macro-structure without internal defects, but with fulfillment of the conditions of directional solidification and compensation for shrinkage.

As a result of the optimization problem solution, the regularity of change in the kinetic operator of the equation is characterized by successive reduction with time in its intrinsic values, which are equated at specified points.

The connection to variational principles dictate a certain ranking for the eigen values of the equation and their interrelation, by virtue of which the optimal model (referred to dimensionless form) is characterized by two parameters: the number of equations n and the variability parameter g, which are common to the macro-model.

The solution of the optimization problem ensures a maximum value of the functional for microstructural and macrostructural ordering. In this case, the optimal dynamics at the boundary of the bar cross section functions of the boundary conditions in calculating the distribution of the physical processes within the solidified bar.

The subsequent calculation procedure consists of identifying the parameters of the synthesized optimal model to the real

Table 1 Main interrelated physical phenomena involved in horizontal continuous casting

Physical phenomena and their notations	Phase-space coordinates (x^l)	Phase-time coordinates (x^t)
1. *TS*	x^1_{n+1} $\int \theta dl \sim \sigma(l)$	x^t_{n+1} $\sigma(t)$
2. *T* (initial phenomenon)	x^1_n $\theta(l) \sim \frac{\partial \sigma}{\partial l}$	x^t_n $\theta(t) \sim \frac{\partial \sigma}{\partial l}$
3. *S* under temperature gradient	x^1_{n-1} $\frac{\partial \theta}{\partial l} \sim v_c$	x^t_{n-1} $\frac{\partial \theta}{\partial t} \sim \Delta C$
4. *D* under heat flow	x^1_{n-2} $\frac{\partial^2 \theta}{\partial l^2} \sim \frac{\partial v_c}{\partial l}$	x^t_{n-2} $\frac{\partial^2 \theta}{\partial t^2} \sim \frac{\partial \Delta C}{\partial t}$
5. *M*	x^1_{n-3} $\frac{\partial^2 \Delta C}{\partial l^2} \sim v_m$	x^t_{n-3} $\frac{\partial^2 \Delta C}{\partial t^2}$
	$\frac{\partial \sigma}{\partial l} \sim \frac{\partial \rho}{\partial t} \sim \frac{\partial \varepsilon}{\partial t}$	
6. *PT* causing stresses and pressure	x^1_{n-4} $\frac{\partial v_m}{\partial l} \sim \frac{\partial^3 \theta}{\partial t^3}$	x^t_{n-4} $\frac{\partial v_m}{\partial t} \sim \int p dl \sim \frac{\partial \rho}{\partial t^2}$
7. *HT*s in liquid-solid state	x^1_{n-5} $\frac{\partial^2 v_m}{\partial l^2} \sim p(l)$	x^t_{n-5} $p(t)$
8. *H* developing a pressure	x^1_{n-6} $\frac{\partial p}{\partial l}$	x^t_{n-6} $\frac{\partial p}{\partial t}$

Note: *TS*, thermomechanical stresses; *T*, heat conductivity; *S*, solification; *D*, effective diffusion; *M*, mass transfer; *PT*, phase transformation; *HT*, hydrodynamic transformation; *H*, hydrodynamic mechanism

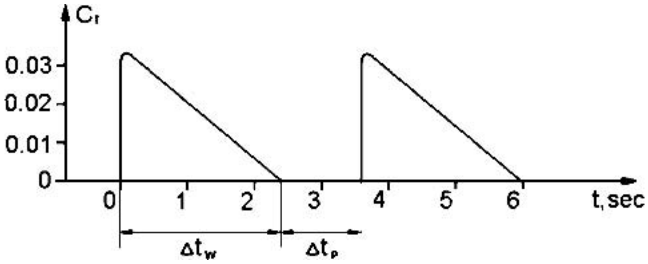

Fig. 2 The computer-generated HCC cycle recommends impulse withdrawing, where Δt_w is the time of drawing and Δt_p is a pause.

casting process parameters (i.e., the HCC of the circular cross section DI bars). This enables one to determine, for the bars of a given grade of cast iron, the model indicator of the process (i.e., the specific Hamiltonian $h_v \sim k_e$) (Ref 3), which is used to calculate the new process parameters and admits a direct measurement.

The systemic solidification macro-model (Table 1) includes the hydrodynamic components (*H*, *HT*) that interact with the motion of the liquid and the solid phases that participate in HCC process.

The feeding and solidification processes in this model are mutually interconnected in such a way that the speed of the liquid metal movement is coordinated with the changing density of the solidified bar. Figure 2 illustrates the computer-generated optimal conditions of "impulse" withdrawing of the bar. Impulse withdrawing means that the drawing speed changes in a discontinuous way and is characterized by a cycle frequency (e.g., drawing-pause-drawing-pause).

Figure 3 shows the optimal feeding parameters $C\ (l_n) = C\ [l_n\ (t_n,\ \Delta l_T,\ \Delta t_c,\ \Delta t)]$ that satisfy conditions where t_n is the starting point of changing the linear feeding velocity, $C(t)$, t_k is the fixed time interval, Δt is the current time interval, Δl_T is the space interval of the impulse feeding controls, and Δt_c is the time interval of stopping the feeding impulses.

The linear bar-size increment satisfies the function $l_n = (C - C_l)t_{n-1}$, where the component:

$$C = \frac{\Delta l_T}{\Delta t}\left(1 - \frac{\Delta t_c}{t_k}\right)$$

characterizes the average speed of feeding, and C_l is the average solidification speed. In this expression, $\Delta t = tk - tn$.

The difference l_n defines the space distribution of metal that is necessary to compensate a volumetric contraction at $l = l_n$ during the time interval t_{n-1}. Under real conditions of the gravitational feeding, the impulse cycle takes place when the continuous speed feeding C_l is equal to the average solidification speed, and the bar height h is equal to l_n.

3. Computation of HCC Parameters

The time for solid skin formation, which is required to start the extraction of bar, or its withdrawal, is governed by the conditions that are needed to control the elastic deformations with compensation for shrinkage. The suggested method and algorithm for its implementation allow the computation of the optimal value of casting speed, the drawing and pause intervals, the cast iron temperature in the receiver, the cast-bar

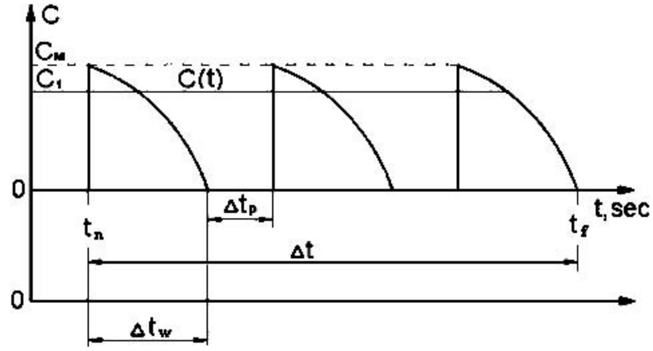

Fig. 3 The optimal feeding conditions (where C_M is the maximal solidification speed and C_l is the average solidification speed) (and $\Delta t_w \sim \Delta t_r$ and $\Delta t_p \sim \Delta t_c$) recommended to compensate for solidification shrinkage

temperature, and the variation in the flow rate and water temperature in the die. Using the model, it is possible to determine the parameters of die design and secondary cooling conditions. Table 2 illustrates the results of the computation of the major technological parameters for continuous-cast, nonround DI bars. Figure 4 shows the results of this computation as a nomogram that can be used to determine the HCC parameters for DI cylindrical bars: 1, for the common usage parts; and 2, for the hydraulic parts.

4. Computation of Chemical Compositions and Microstructure Prediction

For the calculation of the chemical composition of continuous-cast DI bars, the following considerations have been taken into account. Each cross section of the continuous-cast bar is characterized by its own solidification pipe with a zone of effective diffusion (Fig. 1). The concentration of chemical components in this zone depends on the solidification time. With increasing cross-sectioning of the bar, the solidification time and the time of effective diffusion increase, resulting in structural microheterogeneity. To ensure the same concentration of chemical elements in the model bar as that in the actual bar, the model controls the concentration gradient at the boundary of the solidification zone and its distribution within this zone.

The first problem is solved by selecting the chemical composition of the bar as a function of the distribution of chemical elements of the initial concentration. Knowing the change in concentration of chemical elements during the solidification of the base bar, the optimal chemical composition of the bar is computed.

The model also calculates the amount of alloying elements, for example, Sn, Cu, and Mo, that is required under real solidification conditions to control the coefficient of the initial distribution within the zone of effective diffusion to provide the desired as-cast microstructure.

With the mathematical model, the precipitation of new phases can be determined, particularly the nodule count (NC) at the points of overlapping of phenomena, which are analogs to phase conversion. The model calculates the phase transformation in the presence of k points of nonequilibrium (i.e., it specifically counts the NCs when the elements number $[n - 4 + k]$ or two element numbers $[n - 2 + k]$ are precipitated).

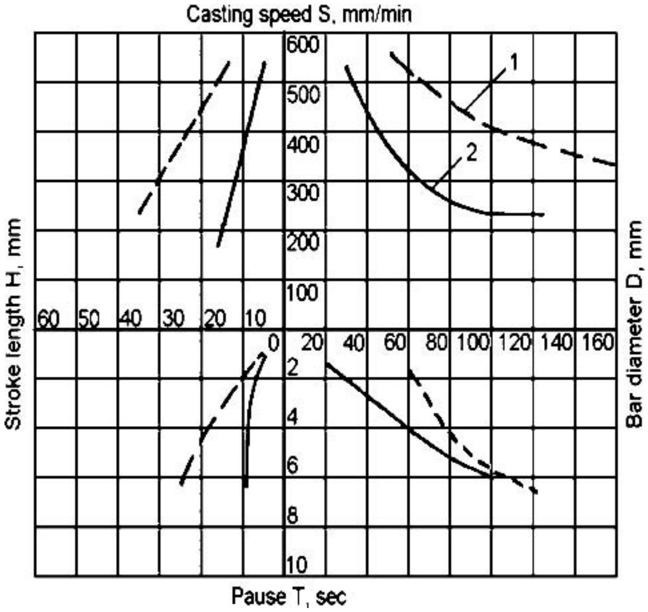

Fig. 4 Chart to compute parameters of the HCC of round DI bars: 1, for common usage parts; 2, for hydraulic parts.

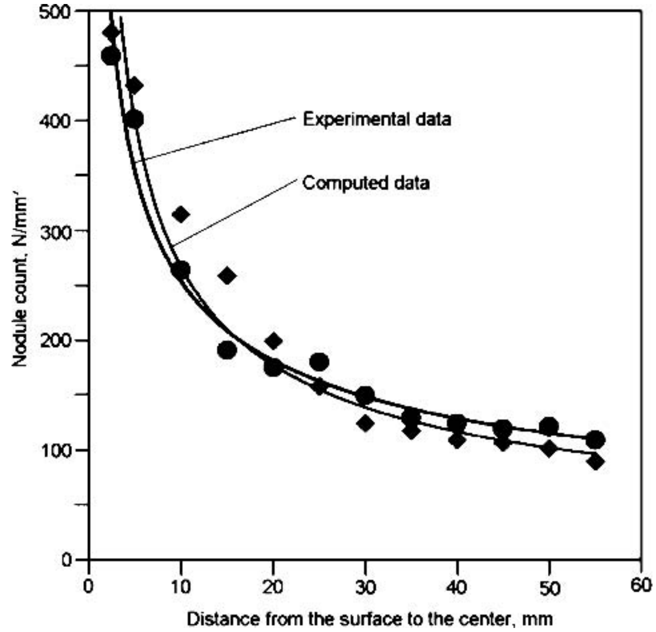

Fig. 5 The actual and calculated distributions of the NC at a distance (*L*) from the edge of a 110 mm DI bar that is intended for an hydraulic pump housing

Table 2 Computed parameters of horizontal continuous casting of nonround ductile iron bars

					Stroke length, mm		Pause, s		Temperature, °C	
Grade of ductal iron in ASTM A-536	Cross-section size, mm	Cross-section area, mm²	Casting speed, mm/min	Drawing speed, mm/s	Bars for common application	Bars for hydraulic valves	Bars for common application	Bars for hydraulic valves	Of bar at outlet from die	In iron receiver
80-55-06	86 × 52	4,442	250.68	12.68	35.9	14.2	5.7	2.2	899	1263
65-45-12	174 × 144	25,056	155.04	7.75	58.1	24.9	14.9	6.4	950	1244
80-55-06	51 × 41	2,091	301.14	15.06	29.9	11.8	3.9	1.5	880	1277
65-45-12	70 × 46	3,220	274.8	13.74	32.7	14.0	4.7	2.0	890	1266

The radius *R* of the bar corresponds to the model length of the solidification pipe, which is characterized at a fixed angle at its vertex by a segment of the helix of the cone $L(n + 1)$ in which the number of elements, defined by the ratio:

$$\left(\frac{L(n+1)}{L(n+k-1)}\right) = 2\,\frac{(n-2-k)}{(1+g^2)^2}$$

is found.

Hence, the *NC* per unit of cross-sectional area $S = \pi R^2$ precipitated at the *k*th point is determined as:

$$NC = \frac{4\pi}{S(n-2+k)^2\left(\dfrac{L(n+1)}{L(n+k-1)}\right)^2(1+g^2)^2} \qquad \text{(Eq 1)}$$

where *n* and *g* are the parameters of the optimal model, $L(n + 1) = L(n,g)$ is the model length of the solidification pipe, and $L(n - 4) = L(n,g)$ is the model length of the effective diffusion zone.

Figure 5 shows the actual (curve 1) and calculated (curve 2) distribution of *NC*s at a distance *L* from the edge of the 100 mm diameter DI bar that was intended for use as a hydraulic pump housing. This method permits the determination of the *NC* at any point along the entire cross section of the bar with good accuracy.

The IMD model and the regular thermal two-phase zone model were compared employing the numerical simulation for the same bars. The results of this comparison show that the parameters calculated using the new model agree better with practical data. A special feature of the IMD model is that the intervals of withdrawal and pauses are determined on the basis of the model rather than being set externally as in the conventional approach.

The HCC parameters calculated on the basis of the model were verified under industrial conditions during the production of DI bars, and good agreement of the modeling results with experimental data were confirmed.

5. The Automatic Control System Development

Experiments have been conducted using the mathematical model that was developed as an integrated, computerized, self-adaptive module to control real HCC parameters that were preset using the data in Table 2. An HCC machine installed in a foundry was equipped with a series of thermocouples to monitor and record the temperature of the iron in the receiver, the cast bar temperature, and the water temperature in the die. A special device controlled the water flow rate in the die cooling system. The data were automatically input into the computer using software that was able to adjust and to simultaneously and automatically control the drawing parameters (i.e., the speed and intervals spacing) with respect to the actual process variables.

A successful series of experiments have confirmed the ability of the solidification model to simulate and control the casting process. The systematic mathematical informational approach has also been successfully used for the modeling of the permanent mold casting of DI and for the calculation of feeding systems (Ref 8).

6. Conclusions

A new approach to the simulation and control of the solidification process using an IMD approach has been described. This new technique has been applied to the HCC of DI. A comparison to actual test runs validated the accuracy and usefulness of this new approach.

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References

1. B. Goodell, Continuous Casting of Ductile Iron: A Numerical Approach, *AFS Trans.,* (Vol 95), 1987, p 613-616
2. Y.P. Zhang and J.Y. Su, Modeling on Solidificatioin Process of Horizontal Continuous Casting of Round Iron Bars, *AFS Trans.,* Vol 109 (No. 01-110 P 1-5), 2001, p 1-5
3. V. Lerner, Mathematical Foundation of Information Macrodynamics, *Int. J. Syst. Anal. Model. Simulat.,* Vol 26, 1996, p 119-184.
4. V. Lerner, *Variation Principle in Informational Macrodynamics,* Kluwer Academic Publishers, 2003
5. C. Eabu, Properties of Continuous Cast Austempered Ductile Iron Bar, *2nd Int. Conf. on Austempered Ductile Iron: Your Means to Improve Performance, Productivity and Cost,* ASME, 1985, p 215-226
6. Y. Lerner and G. Griffin, Developments in Continuous Casting of Gray and Ductile Iron, *Mod. Cast.,* 1997, p 41-44
7. Y. Lerner, Continuous Casting of Ductile Iron. Solidification, Microstructure, and Properties, *ISS 50th Electric Furnace Conf. Proc.,* Vol 50, 1992, p 331-340
8. V. Lerner and Y. Lerner, A New Approach to the Solidification Modeling of Casting Processes, *Proc. 1st Int. Conf. on Mathematical Modeling of Metalworking Processes,* MMT, 2000, p 351-360